APPLICATION NO.   09/826,118

TITLE OF INVENTION: Wavelet Multi-Resolution Waveforms

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Currently amended CLAIMS

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CLAIMS

10   WHAT IS CLAIMED IS:

Claim 1. (deleted)

Claim 2. (deleted)

Claim 4. (deleted)

15   Claim 5. (deleted_

Claim 6. (deleted)

Claim **7**. (new)   A method for implementing nother Wavelet
waveforms and filters for communication applications,~~An iterative~~
20   ~~eigenvalue least-squares LS   digital mother Wavelets at baseband~~
~~for means for the design of new multi-resolution waveforms and~~
~~filters,~~ said method comprising steps:
said mother Wavelet $\psi(n)$ with sample index n is a digital finite
      impulse response (FIR) waveform at baseband (zero frequency
25   offset) in the time domain,
requirements for linear phase FIR filters are specified by the
      passband and stopband performance of the power spectral
      density (PSD) which requirements are incorporated into
      quadratic error metrics J(pass), J(stop),
30   ~~power spectral density PSD representative requirements for said~~
      ~~mother Wavelet ψ frequency ω response $\psi_\omega(\omega)$ in a multi-~~
      ~~channel filter bank,   specify~~
      ~~a) passband frequency range for waveform transmission,~~
      ~~b) stopband spacing between adjacent filters,~~
35   ~~c) bounds on ripple over said passband,~~

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d) stopband filter attenuation,

e) rolloff with frequency outside stopband,

f) quadrature mirror filters QMF require the sum of said

PSD's for contiguous filter responses to be flat over

deadband which is said stopband,

g) symbol-to-symbol interference ISI,

h) adjacent channel interference ACI,

said LS error metrics to measure said requirements (a)-(h) are

derived as functions of said Wavelet $\psi(n)$ assuming

i) T and 1/T are sample interval and sample rate equal to

Nyquist sample rate,

j) ψ is real and symmetric about n=0,

k) n= 0, +/-1, . . ., +/-ML/2 digital index over said ψ,

l) M is interval between contiguous said ψ,

m) 1/MT is said ψ symbol rate and channel-to-channel

separation,

n) L is length of said ψ in units of said M,

said multiple-resolution properties require said LS metrics

to be constructed as functions of said Wavelet Fourier

harmonics $\psi_k(k)$ with k=0, +/-1, . . ., +/-(N_k-1) and

$N_k \geq L$ is a design parameter,

it is sufficient to use positive n=0,1, . . ., ML/2 and k=0,1, . .

., N_k-1 since said $\psi(n)$ and $\psi_k(k)$ are real and symmetric,

ML/2+1 x N_k matrix bw wherein "x" reads "by" maps $\psi_k(k)$ into

$\psi(n)$ to within a scale factor by equations

$\psi(n) = \sum_k bw(n+1,k+1)\psi_k(k)$ for n≥0, k≥0,

bw(n+1,k+1) = 1   for n=0,

= 2 cos(2π nk/ML) otherwise,

= row n+1, column k+1 element of bw,

Wavelet requirements on the deadband for quadrature mirror

filter (QMF) properties for perfect reconstruction are

incorporated into the quadratic error metric J(dead),

Wavelet orthogonality requirements are expressed by the error

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metrics J(ISI), J(ACI) for intersymbol interference (ISI) and adjacent channel interference (ACI) which are non-linear quadratic error metrics in said FIR ψ(n) used to control said ISI and ACI levels,

5 Wavelet multi-resolution property requires said quadratic error metrics to be converted to quadratic error metrics in the discrete Fourier harmonics ψ(k) of said ψ(n) wherein k is the harmonic index,

eigenvalue algorithm requires the error metrics to be linear

10 quadratic forms in the ψ(k) and finds the eigenvectors equal to the ψ(k) coefficients which minimize the weighted sum J of said quadratic error metrics,

step 1 of the iterative algorithm implements said eigenvalue algorithm to find said optimum ψ(k) for the weighted sum

15 sum of J(pass), J(stop), J(dead),

step 2 linearizes said J(ISI),J(ACI) with said ψ(k) from step 1,

step 3 uses said eigenvalue algorithm to find said optimum ψ(k) for the sum J of J(pass), J(stop), J(dead) and the linearized J(IXI),J(ACI),

20 step 4 checks to see if said iteration has converged,

step 5 returns to step 2 if said iteration has not converged and linearizes said J(ISI),J(ACI) with said ψ(k) from step 4, and stops iteration if said iteration converges,

said ψ(k) from said iteration algorithm is the optimum least-

25 squares error solution that minimizes said J,

use inverse discrete Fourier transform of said ψ(k) to calculate ψ(n) which minimizes J,

use said ψ(n) for the transmitted data symbol waveform in the communications transmitter and,

30 use complex conjugate of said ψ(n) for the impulse response of the detection filter in the communications receiver to remove the received ψ(n) and recover said transmitted data symbols.
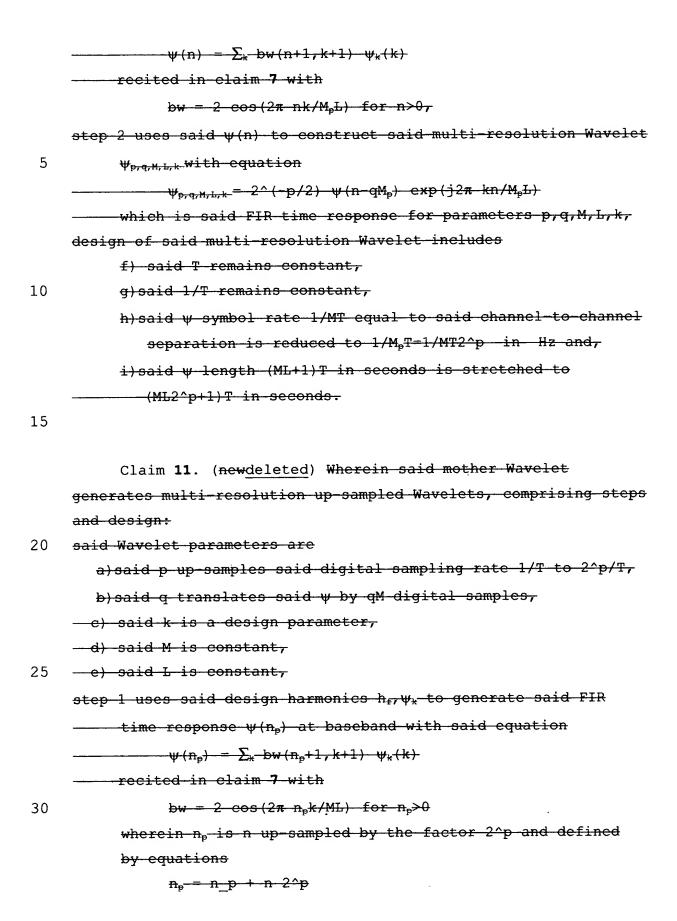
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said LS error metrics are converted by said bw mapping into
quadratic forms in said h_k equal to $J(band) = h_k' R\ h_k$ wherein
said h_k' is the transpose of h_k and said R is a real square
symmetric matrix of LS errors in meeting said requirements,

5　LS ISI,ACI error metrics $J(ISI), J(ACI)$ are derived as non-linear
quadratic forms in h and converted by said bw matrix to the
non-linear quadratic form in h_k equal to $J(ISI) = \delta E' \delta E$,
$J(ACI) = 2\delta E' \delta E$　wherein $\delta E = AH h_k$ is a column vector and
matrix "A" in the matrix product AH is a function of said h

10　hereby introducing said non-linearity, and said AH differ
for ISI and ACI error metrics,
LS cost function J is the weighted sum of said LS error metrics
$$J = \sum w(LS\ metric)\ J(LS\ metric)$$
with summation over said LS metrics= passband, stopband,

15　QMF deadband, ISI, ACI with normalized weights
$$\sum w(LS\ metric) = 1,$$
said weights are free design parameters,
said iterative eigenvalue LS algorithm at each step finds the
optimum eigenvalue and eigenvector which minimize said

20　quadratic form J in h_k for a constant said "A",
said eigenvector is the optimum h_k which minimizes said J and
said bw equation derives the corresponding optimum h which
minimizes said J,
step 1 in said iterative algorithm finds said optimum eigenvalue,

25　eigenvector, h_k, h of J reduced by deleting said non-linear
ISI and ACI LS quadratic error metrics,
said h is used to evaluate said "A" matrices for step 2,
step 2 finds said optimum eigenvalue, eigenvector, h_k, h for
minimum J using said "A" from step 1,

30　said h is used to evaluate said "A" for step 3,
steps 3,4, etc. continue until said minimum J converges to a
steady value and,
said optimum $\psi_k(k)$ uses said bw to calculate optimum $\psi(n)$ for
implementation as said Wavelet FIR digital waveform and

~~filter time response.~~

Claim **8**. (~~new~~currently amended)   A second method for
implementing mother Wavelet waveforms and filters for
communication applications, ~~An LS method for designing digital~~
~~mother Wavelets at baseband for multi-resolution waveforms and~~
~~filters,~~ said method comprising steps:
construct said error metrics J(pass),J(stop), J(dead),J(ISI),
    J(ACI) as quadratic error metrics in ψ(k) as depicted in
    claim 7 and convert these quadratic forms to norm-squared
    error metrics in ψ(k) for least-squares gradient solution
    and construct J as their weighted sum,
step 1 calculates an initial estimate ψ(k) of said solution using
    the Remez-Exchange algorithm for the weighed sum of
    J(pass), J(stop) represented as quadratic error metrics in
    ψ(k),
~~said PSD waveform representative requirements and assumptions~~
    ~~are recited in (a)-(n) in Claim 7,~~
~~said multiple-resolution properties require said LS metrics~~
    ~~to be constructed as functions of said $\psi_k(k)$,~~
~~said LS error metrics for said passband, stopband, and QMF~~
    ~~deadband requirements are derived as squared vector norm~~
    ~~functions of said h and converted by said bw matrix into~~
    ~~J(band) = $\left\|Bh_k\right\|^2$ wherein $\left\|Bh_k\right\|$ is the vector norm of the~~
    ~~column vector Bh_k and said B is the matrix of LS errors in~~
    ~~meeting said requirements and wherein said squared vector~~
    ~~norm is suitable for LS optimization,~~
~~LS ISI,ACI error metrics J(ISI),J(ACI) are derived as squared~~
    ~~vector norm functions equal to J(ISI=$\left\|\delta E\right\|^2$, J(ACI)=2$\left\|\delta E\right\|^2$~~
    ~~using said column vectors $\delta E = AHh_k$ in claim 7,~~
~~LS cost function J is said weighted sum of said LS error metrics~~

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~~equal to J=∑w(LS metric)J(LS metric) defined in claim 7,~~

~~an LS gradient search algorithm finds optimum $h_k(k)$ to~~
~~minimize J,~~

~~step 1 of said LS gradient search algorithm uses a Remez~~

5 ~~exchange algorithm to find said optimum $h_k(k)$ for said J~~
~~reduced to said passband and stopband LS metrics,~~

~~step 2 uses the estimated $h_k(k)$ from step 1 to initialize said~~
~~gradient search,~~

step 2 uses said estimate ψ(k) from step 1 to initialize said

10   gradiant algorithm,

step 3 selects one of several available gradient search
     algorithms, gradient search parameters, and stopping rules,

~~step 4 implements said algorithm, parameters, and stopping rule~~
~~selected in step 3 to derive said optimum $h_k(k)$ to minimize~~

15 ~~J and,~~

step 4 implements said algorithm, parameters, and stopping rule
     selected in step 3 to derive said optimum ψ(k) to
     minimizee J equal to the weighted sum of the norm-squared
     error metrics J(pass), J(stop), J(dead), J(ISI), J(ACI),

20 ~~said optimum h_k uses said bw to calculate optimum ψ(n) for~~
~~implementation as said Wavelet FIR digital waveform and~~
~~filter time response.~~

use inverse discrete Fourier transform of said ψ(k) to calculate
     ψ(n) which minimizes J,

25 use said ψ(n) for the transmitted data symbol waveform in the
     communications transmitter and,

use complex conjugate of said ψ(n) for the impulse response of
     the detection filter in the communications receiver to
     remove the received ψ(n) and recover said transmitted data

30   symbols.

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Claim **9**. (~~new~~deleted) ~~Wherein said mother Wavelet generates~~
~~multi-resolution dilated Wavelets, comprising steps and design:~~
~~said Wavelet parameters are~~

~~a) scaling parameter p dilates sampling by factor 2^p~~
~~equivalent to sub-sampling by factor 2^p,~~
~~b) translation parameter q translates said ψ by qM digital~~
~~samples,~~
~~c) frequency offset k is set by design,~~
~~d) symbol repetition interval said M remains constant,~~
~~e) Wavelet length said L in units of said M remains~~
~~constant,~~
~~step 1 uses said design harmonics ψ_k to generate said FIR time~~
~~response ψ(n_p) at baseband with said bw equation~~
~~ψ(n_p) = Σ_k bw(n_p+1,k+1) ψ_k(k)~~
~~recited in claim 7 with~~
~~bw = 2 cos(2π n_p k/ML) for n_p>0~~
~~wherein n_p=n/2^p is n sub-sampled or equivalently dilated~~
~~by the factor 2^p,~~
~~step 2 uses said ψ(n_p) to construct said multi-resolution~~
~~Wavelet ψ_{p,q,M,L,k} with equation~~
~~ψ_{p,q,M,L,k} = 2^(-p/2) ψ(n_p-qM) exp(j2π kn_p/ML)~~
~~which is said FIR time response for parameters p,q,M,L,k~~
~~wherein the subset p,M,L are the scale parameters,~~
~~design of said multi-resolution Wavelet includes~~
~~f) said T for n is increased to T2^p for n_p,~~
~~g) said 1/T is reduced to 1/T2^p,~~
~~h) said ψ symbol rate 1/MT equal to said channel-to-channel~~
~~separation is reduced to 1/MT2^p in Hz and,~~
~~i) said ψ length (ML+1)T in seconds is stretched to~~
~~(ML+1)T2^p in seconds.~~

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Claim **10** (~~new~~currently amended) A method for implementing
Wavelet waveforms and filters for multi-resolution communication
applications derived from said mother Wavelets in claims 7 or 8,
comprising steps:~~Wherein said mother Wavelet generates multi-~~

5 ~~resolution constant sample rate dilated Wavelets, comprising~~
~~steps and design:~~
said mother Wavelet is designed for application to an M channel
        polyphase filter bank as depicted in claims 7 or 8 wherein
        M is the spacing between Wavelets within said channels for

10   the Nyquist digital filter bank sample rate 1/T,
said multi-resolution changes the number of said user channels
        to M2^p while keeping the same channel filter design which
        means said Nyquist digital sample rate is changed to
        2*p/T wherein Wavelet scale parameter p is an integer,

15 said multi-resolution Wavelet FIR ψ(n) is derived from said
        mother Wavelet design harmonics ψ(k) using the inverse
        discrete Fourier transform for the mapping of ψ(k) to ψ(k),
use said ψ(n) for the transmitted data symbol waveform for each
        transmit channel in the communications transmitter and,

20 use complex conjugate of said ψ(n) for the impulse response of
        the detection filter bank in the communications receiver
        which is used to remove the received ψ(n) and recover said
        transmitted data symbols.
~~said Wavelet parameters are~~

25   ~~a)said p dilates said ψ to increase said length from~~
        ~~ML+1 to M$_p$L+1 where M$_p$=M2^p is the dilated interval~~
        ~~between contiguous ψ's,~~
        ~~b)said q translates said ψ by qM$_p$ digital samples,~~
        ~~c)said k is set by design,~~

30   ~~d)said M$_p$=M2^p is dilated M,~~
        ~~e)said L remains constant,~~
~~step 1 uses said design harmonics ψ_k to generate said FIR time~~
        ~~response ψ(n$_p$) at baseband with said bw equation~~

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$$\psi(n) = \sum_k bw(n+1,k+1) \psi_k(k)$$

recited in claim 7 with

$$bw = 2 \cos(2\pi \, nk/M_pL) \text{ for } n>0,$$

step 2 uses said $\psi(n)$ to construct said multi-resolution Wavelet

5      $\psi_{p,q,M,L,k}$ with equation

$$\psi_{p,q,M,L,k} = 2^{\wedge}(-p/2) \, \psi(n-qM_p) \, \exp(j2\pi \, kn/M_pL)$$

which is said FIR time response for parameters p,q,M,L,k,

design of said multi-resolution Wavelet includes

     f) said T remains constant,

10   g) said 1/T remains constant,

     h) said ψ symbol rate 1/MT equal to said channel-to-channel

         separation is reduced to $1/M_pT=1/MT2^{\wedge}p$ in Hz and,

     i) said ψ length (ML+1)T in seconds is stretched to

     $(ML2^{\wedge}p+1)T$ in seconds.

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Claim **11**. (newdeleted) Wherein said mother Wavelet
generates multi-resolution up-sampled Wavelets, comprising steps
and design:

20 said Wavelet parameters are

     a) said p up-samples said digital sampling rate 1/T to $2^{\wedge}p/T$,

     b) said q translates said ψ by qM digital samples,

     c) said k is a design parameter,

     d) said M is constant,

25   e) said L is constant,

step 1 uses said design harmonics h_f, ψ_k to generate said FIR

     time response $\psi(n_p)$ at baseband with said equation

$$\psi(n_p) = \sum_k bw(n_p+1,k+1) \psi_k(k)$$

recited in claim 7 with

30      $bw = 2 \cos(2\pi \, n_p k/ML) \text{ for } n_p>0$

wherein n_p is n up-sampled by the factor $2^{\wedge}p$ and defined
by equations

     $n_p = n\_p + n \, 2^{\wedge}p$

~~n_p = 0,1,2,. . .,2^p-1~~

~~wherein n_p is the index over the additional samples added~~
~~to each sample n by said up-sampling,~~
~~step 2 uses said $\psi(n_p)$ to construct said multi-resolution Wavelet~~
~~$\psi_{p,q,M,L,k}$ with equation~~

~~$\psi_{p,q,M,L,k} = 2^{(-p/2)} \psi(n_p-qM) \exp(j2\pi kn_p/ML)$~~
~~which is said FIR time response for parameters p,q,M,L,k,~~
~~design of said multi-resolution Wavelet includes~~
~~f) said T is decreased to T/2^p,~~
~~g) said 1/T is increased to 2^p/T,~~
~~h) said ψ symbol rate 1/MT equal to said channel-to-channel~~
~~separation is increased to 2^p/MT in Hz and,~~
~~i) said ψ length (ML+1)T in seconds is reduced to~~
~~(ML +1)T/2^p in seconds.~~

Claim **12** (~~new~~<u>currently amended</u>) Wherein said multi-resolution Wavelets have properties comprising:

<u>said multi-resolution Wavelets $\psi(n)$ at baseband are derived from</u>
<u>said mother Wavelet using said design harmonics $\psi(k)$ and</u>
<u>scale parameters said dilation p, said number of samples M</u>
<u>over Wavelet spacing, length (L) in units of M, said</u>
<u>digital sample rate 1/T, and translation parameter.</u>
<u>said $\psi(n)$ can be designed to support a bandwidth(B)-time(T)</u>
<u>product BT=1+α with no excess bandwidth $\alpha=0$,</u>
~~said scale parameters p,M,L and said design parameter 1/T specify~~
~~said multi- resolution Wavelets at baseband and said q,k~~
~~specify time, frequency translations from baseband,~~
~~said design harmonics $\psi_k(k)$ of mother Wavelet are said design~~
~~coordinates for multi-resolution Wavelets,~~
~~said design harmonics $\psi_k(k)$ use said bw matrix to generate said~~
~~multi-resolution Wavelet baseband time response $\psi(n)$ for~~
~~said dilation, dilation of Wavelet length, and up-~~

~~said design harmonics $\psi_k(k)$ are few in number compared to said~~

5   ~~$\psi(n)$,~~

~~said ψ is designed to support a bandwidth-time product $B_f T=1+\alpha$ with no zero excess bandwidth $\alpha=0$,~~

said $\psi(n)$ can be designed to support a bandwidth(B)-time(T) product BT=1+α with no excess bandwidth $\alpha=0$,

10   said multi-resolution Wavelets are designed to behave like an accordion in that at different scales said Wavelets are stretched and compressed versions of the mother Wavelet with appropriate time and frequency translation,

~~said optimization techniques in claims 7,8 assume said $\psi(n)$~~

15   ~~symmetric about n=0 and are applicable to other arrangements of $\psi(n)$ with self-evident modifications,~~

~~optimization algorithms for finding said optimum set of $\psi_k(k)$ use said linear LS waveform and filter design methods recited in claims 7,8 and also use other methods and,~~

20   said linear waveform and filter ~~LS~~ least-squares design methods can be modified to design waveforms for other applications including bandwidth efficient modulation ~~BEM~~ and synthetic aperture radar ~~RAR.~~ and,

other optimization algorithms exist for finding said optimum

25     $\psi(n)$.

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